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## **Doctoral Thesis**

D. H. Keunig: On the theory of incomplete, piezoelectric bimorphs with experimental verification,\*

Thesis University of Groningen, 1970, 120 pages.

In this thesis the linear, three-dimensional, piezoelectric equations are reduced to approximate, two-dimensional ones, treating the static and dynamic flexure of thin bimorphs, partly coated by electrodes (incomplete bimorphs). For that purpose two-dimensional equations are derived for piezoelectric plates and bimorphs with completely coated faces. An important assumption about the charge distribution on the inner electrode of incomplete bimorphs is given, stating that the charge vanishes in those parts where the outer faces are free of electrodes. This assumption allows the application of the mentioned, approximate equations for plates and bimorphs to the parts of the incomplete bimorphs. In this way an approximate theory for these bimorphs is obtained.

The approximate theories are checked by experiments. Experimental data are obtained for circular, piezoceramic plates, bimorphs and incomplete bimorphs. The plate is loaded by a singular force acting at the centre. The potential difference between a series of points on the faces and the middle plane is measured. The bimorph is loaded by an eccentric, singular force. The resulting potential difference between the outer electrodes and the inner one is measured for a series of values of the distance between load and centre. The outer faces of the incomplete bimorph are successively covered by a number of concentric, circular electrodes with increasing radius; this bimorph is loaded by a singular force at the centre. The potential difference between the shorted, outer electrodes and the inner one is measured as a function of the radius of the outer electrodes.

The assumption concerning the charge distribution on the central electrode of incomplete bimorphs is verified by calculating this distribution from the exact, three-dimensional equations for an infinite, incomplete, piezoceramic bimorph. This bimorph is deformed by a potential difference between the inner and outer electrodes. Also the charge distribution on the outer electrodes and the potential on the faces are computed. It appears that these computed quantities tend to the values obtained from the approximate treatment of this problem as the ratio total thickness over diameter outer electrodes (h) tends to zero. For  $h \le 0.2$  the exact and approximate values agree in the intervals  $(0, 1-h)(1+2h, \infty)$  within a deviation less than 5%.

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